

## Performance Analysis of Polar decoders for 5G Technology

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**Abstract:** The term "5G" refers to the next generation of mobile communication networks, which represents a significant advance in wireless communication. Much further advancements in mobile communications are made by 5G technologies. Recently, there has been much debate in academia and industry over potential channel coding methods for fifth-generation (5G) mobile communication systems. Among other codes, polar codes have caught researchers' interest due to their capacity-achieving qualities. The polar encoding system's fundamental operation, channel polarization, makes it possible to achieve channel capacity. In this research, we provide polar code encoding and decoding and analyze the performance of different decoders for short to intermediate information length polar codes. The Bit Error Rate (BER) and Frame Error Rate (FER) of the decoders for various bits are considered in the proposed research.

**Keywords:** Successive Cancellation (SC), Simplified Successive Cancellation (SSC), Cyclic Redundancy Check-Aided SCL (CA SCL), Belief Propagation (BP) decoder and Soft Cancellation (SCAN)

### 1. Introduction

The year 1948 [1] signifies the beginning of classical coding theory, which has been introduced by Claude Shannon's presentation of the concept of 'channel capacity'. By using error correction codes with coding rates lower than the channel capacities and codeword lengths that are infinitely long, The seminal study of Shannon [1] transmission over noisy channels was supposed to be nearly error-free. According to the Shannon-Hartley theorem, the following method computes bandwidth B, An AWGN channel has a bandwidth of B Hz, and its noise power spectral density is  $N_0/2$  Watts per Hz for each dimension.

$$C = B \log_2 \left( 1 + \frac{S}{N_0 B} \right), \quad (1)$$

S Watts is the average transmitted power. Therefore, the error correction codes will only be encoded at a maximum coding rate is constrained by the Signal-to-Noise Ratio which is denoted by the notation SNR . Therefore, the Signal-to-Noise Ratio , which is represented by the notation, places a limit on the upper limits rate of error - correcting code and code In a similar manner, the capacities of a BEC (Binary Erasure Channel) or a BSC (Binary Symmetric Channel) can be characterised by their individual channel characteristics, which include. which depend on the type of channel being used. Assuming limitless processing and time resources, i.e., the chance of the BSC and BEC crossing over and eliminating each other. In practice, no practical system can support either an infinitely elaborate implementation or an unlimited transmission delay. So, in order to attain the intended performance metrics, we need optimised codes that run close to the Shannon's capacity limit, as depicted in Figure. 1.

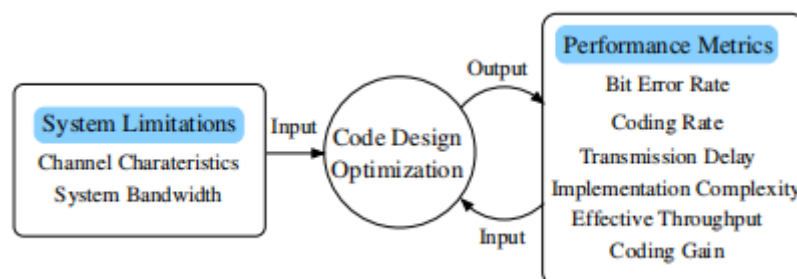
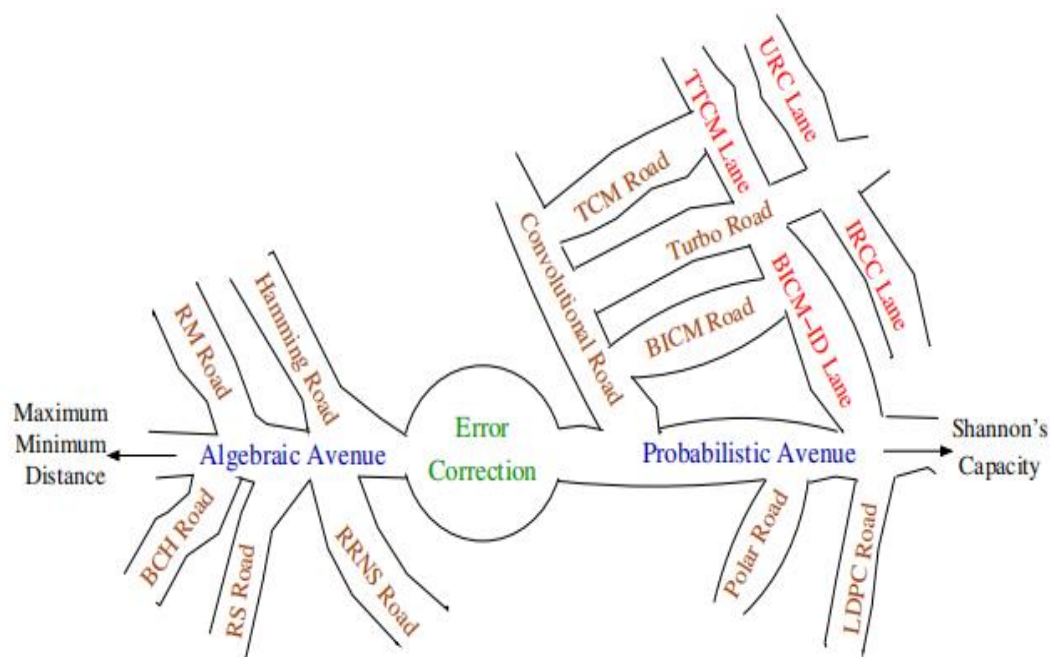


Figure 1 : Factors driving the code design optimization

On the basis of random coding, Shannon measured the maximum capacity and established the existence of 'capacity-achieving' codes. However, he didn't provide any instructions on how to build such scripts. Scientists have been working for about the last 70 years on creating the best possible codes that can operate at or near their limit of capacity while still providing the necessary performance metrics shown in Figure 1. In broad terms, there are two ways to look at this investigation: Algebraic and probabilistic coding avenues depicted on the stylised road map of Figure. 2



**Figure 2 : Classical channel coding theory's progress is depicted in this timeline. Increasing the lowest distance is the goal of the algebraic technique; increasing capacity is the goal of the probabilistic approach.**

Research in the first couple decades focused mostly on algebraic coding. For each given coding rate, this coding paradigm seeks to maximize the minimal Hamming distance<sup>1</sup> between the code words, or to be more precise, for every information word length and any code word length  $k$  and any code word length  $n$ , in order to create powerful codes. Many different families of codes are now in use, such as the Hamming and Reed-Muller (RM) and (RM) codes, the BCH (Bose-Chaudhuri) and (BCH) codes, the Reed Solomon (RS) codes, and the RRNS (Redundant Residue Number System) codes, to name a few. A capacity-achieving design cannot be guaranteed by using the algebraic route. Mathematical codes have made it into real-world use, however, because of the robustness and reduced BER (bit error rate) of their error correction. Specifically, Codes like these come in handy when faced with difficult choices. Since RS codes serve as an outer layer<sup>2</sup> of error checking (sometimes referred to help lower the BER floor in magnetic tape and disc storage due to the internal layer of error checking, they are commonly utilized in these types of standardized systems also known as the inner code. The probabilistic coding path has opened the door to capacity, in contrast, the avenue of algebraic coding is open. Probabilistic coding is used in Figure 2. explicitly based on Shannon's random coding theory and aims to balance performance and complexity in a fair manner. Convolutional codes [2], Low Density Parity Check codes[3][5], turbo codes [6], [7], and polar codes [8] were all constructed along this path. Such as turbo BCH codes (e.g., turbo Hamming codes [9], IRCC (Irregular Convolutional Code) assisted concatenated schemes) [10, 12] and turbo Unity Rate Code-assisted are also included in the probabilistic coding paradigm, along with the coded modulation schemes, such as Trellis Coded Modulation (TCM), Bit-Interleaved Coded Modulation (BICM), Bit-Interleaved Coded Modulation with Iterative Decoding, and Turbo BICM. Turbo and Low-Density Parity Polar codes

finally made it able to operate approximately close to the Shannon limit and proved to be having the potential to reach the necessary level of capacity, albeit at innumerable codeword lengths. For the past two decades, turbo and LPC codes have been the dominant codes, but polar coding has shown itself to be a serious challenge to their dominance. When it comes to the eMBB (enhanced mobile broadband) and in addition to the physical broadcast channel, Polar codes are already a part of the 5G New Radio platform (PBCH) thanks to their early adoption. Data and control channels using polar codes have been identified in mMTC, Massive Machine Type Communication use cases. At times, polar codes arose whenever the apprehension that "coding is dead" reappeared. By finding polar codes, scientists were given new impetus and an entirely new approach for reaching Shannon's theoretical limit. Polar codes have also received a great deal of interest in quantum theory. In response to the growing interest in polar codes, this report gives a detailed analysis of classical and quantum polar codes. We lead to a greater understanding through important historical the encoding and decoding methods related with these innovations will be covered in a methodical way in this course.

The main objectives of this proposed research is to:

- To construct a capacity achieving code for a Binary Discrete Memoryless Channel.
- To achieve an overly small error probability or Bit Error Rate, if you code properly.

The related works in relation to the decoders in the literature are dealt in section 2. Section 3 deals with the various decoder techniques like Successive Cancellation (SC), Simplified Successive Cancellation (SSC), Cyclic Redundancy Check-Aided SCL (CA SCL), Belief Propagation (BP) decoder, and Soft Cancellation (SCAN) used in the research for improving capacity is discussed in detail. The 4<sup>th</sup> section deals with the BER and FER results obtained for each of these decoders. The fifth section gives the conclusion of the research done on the decoders.

## 2. Literature Review

In the years that followed their invention, polar codes attracted a great deal of interest from researchers working in the fields of communication theory and hardware design. Within this part, the papers that are most pertinent to this thesis will be discussed.

In the beginning, polar codes were solely designed to work with the binary erasure channel. Later on, Tal and Vardy developed a method to determine the dependability of a polar code's sub-channels for the AWGN channel [13], which was based on a target noise level or signal-to-noise ratio (SNR). In order to develop polar codes that may be used for simulation, this process of construction is used all through this thesis.

The very first hardware architectures for the SC decoding of polar codes were proposed by Leroux et al. [14, 15]. In specific, they described a pipelined architecture that consists of  $N$  minus one processing elements (PE) organized in a tree structure with pipeline registers positioned in-between the levels of the tree. The tree was later reduced to a single stage made of  $N$  PEs, retaining the pipeline registers, but lowering logic use dramatically. This architecture was further refined. Decoding multiple frames at the same instant using idle processing units improved the throughput and logic usage of the decoder, but this design was extended to include a frame-overlapped variation.

After further refinement, Leroux et al. proposed a semi-parallel design [16] that had a greater logic utilization rate at the expense of a slight throughput reduction. To further reduce the decoder's latency by 25%, Mishra et al. also proposed a revolutionary look ahead technique in their first polar code decoder ASIC [18], which they later used to construct the very first decoder ASIC for polar codes. In our second-generation design, we use the enhancement outlined in Section 5.2.

Using a similar look-ahead approach, Zhang et al. [19] reduced decoding time by half by pre-computing all possible outcomes of function  $g$ . They subsequently added frame overlapping [20] to the design, which increased its throughput even further. Neither of these approaches is well-suited to the very lengthy code lengths needed by this thesis since they considerably increase memory consumption.

Though a more efficient and less complex SC implementation was demonstrated in [21] as an alternative to Pamuk's successful construction of a decoder using the belief propagation method.

A two-phase SC decoder was also proposed by Pamuk and Arikan [22]. Currently, this is the most advanced SC decoder for moderate-length cables. On the basis of a multi-level breakup of the decoding network, this decoder uses an internal LLR value recomputation method to reduce the memory requirements of the decoder.

Using a list decoding approach, Tal and Vardy [23] presented structures to enhance polar codes' error-correction performance. Using numerous SC decoders in these systems necessitates a large amount of memory and logic. Thus, the low-complexity implementations described in this thesis are of use to the students. Balatsoukas-Stimming and Burg used the SC decoder of [9] to develop the first hardware list decoder [11]. They also came up with a way to reduce the memory needs of list decoding.

The SSC technique [24], devised by Alamdar-Yazdi and Kschischang to address the limited throughput of polar codes, prunes the decoding graph depending on the location of frozen sub-channels in a particular polar code. [25] This design increases throughput significantly, but only offers (hard) decoded bits at the end, making it not suitable for list decoding purposes. In a later extension of the SSC method, Sarkis and Gross introduced an additional simplification that was based on the maximum likelihood (ML) [25]. This revised technique, which has been given the name ML-SSC, reduces the latency of SC polar decoding to an even lower level, so paving the way for high-throughput polar decoders. For instance, the paper reported a hard-output polar decoder that was based on additional modifications to the ML-SSC algorithm and was capable of reaching throughputs that were greater than one gigabit per second.

A concatenated coding system was presented by Mahdavifar et al. [26], which aims to improve the error correction of polar codes by merging them with interleaved Reed-Solomon codes. Because of the presence of several inner polar decoders inside this architecture, it serves as an excellent focus for the designs that are discussed in this thesis. This approach, in contrast to list decoding, does not call for SISO inner decoders to be implemented. Nevertheless, the concatenated technique can only be used for low-rate codes because of its limitations.

### 3. Different types of codes

#### 3.1 Polar codes

As a family of linear block code, polar codes can be generated with the matrix  $\mathbf{G}_N = \mathbf{F}^{\otimes n}$ , where  $\mathbf{F} =$

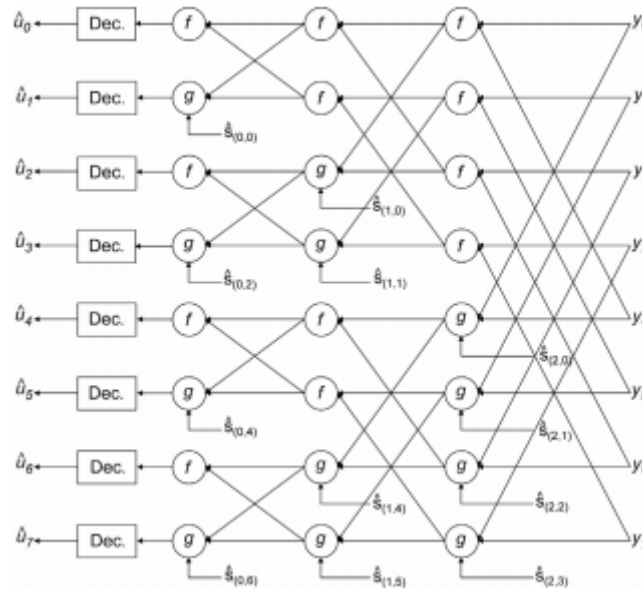
$\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ , and  $N = 2^n$  denotes the block-length. For a given  $N$ , the polar code is obtained by the linear

mapping  $\mathbf{x} = \mathbf{u}\mathbf{G}_N$ , where  $\mathbf{u} = \{u_1, u_2, u_3, \dots, u_N\}$  is the message vector, and  $\mathbf{x} = \{x_1, x_2, x_3, \dots, x_N\}$  is the corresponding codeword. We denote the set of information bits and frozen bits by  $A$  and  $A_c$ , respectively. A polar code  $P(N, K)$  with block-length  $N = 2n$  and coding rate  $K/N$  can be constructed by setting the  $K$ -dimensional set  $\mathbf{u}_A$ ,  $\{u_i : i \in A\}$  with information bits, and froze the rest  $(N - K)$  bits at  $\mathbf{u}_{A_c}$ ,  $\{u_i : i \in A_c\}$  to predetermined values that are known to the decoder, (all frozen bits are assumed to be zeros in this paper if not specified.) In particular, all polar codes in this paper are concatenated with  $r$  CRC bits, thus the actual information rate is  $(K - r)/N$ .

#### 3.2 SC decoding

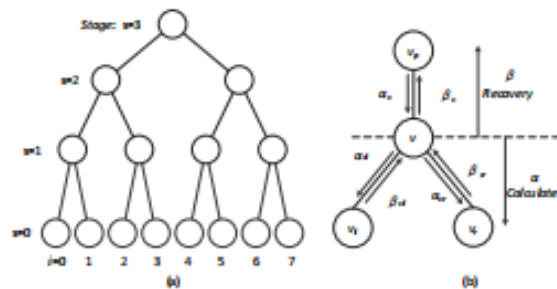
The butterfly units of the polar encoder are responsible for introducing correlation between the source bits. All preceding bits with lower indexes are required to support each subsequent bit with a higher index. When employed, this type of correlation leads to significantly improved decoding performance. As a result, it is the primary notion behind a fundamental decoding technique that is known as SC decoding. The "interference" that is created by the bits that came before is reliably cancelled out bit by bit, making it possible to retrieve source bits with a higher degree of accuracy. Because polar codes have a regular structure, the SC method may be described in terms of an interlocking tree structure. This is because polar codes are sequential. SC decoding can be seen as a soft or hard message transfer mechanism carried out over the lattice of polar code. A  $n$ -layer lattice has  $N$  levels in each row. There are half of  $N$  butterfly units, each of which has two check nodes and two varying ones.

Successive Cancellation decoder will update the messages as they progress through each stage, and it will sequentially decide the estimation bit by bit.



**Figure 1 : Successive Cancellation decoding, N = 8**

Successive cancellation decoding can be represented as a binary tree traversal [6], as shown in Fig. 2(a). Each subtree therein represents a shorter polar code. The set of nodes of the subtree rooted at node  $v$  is denoted by  $V_v$ . Thus  $V_{root}$  denotes the full binary decoding tree. The set of all leaf nodes is denoted by  $U$ . Meanwhile, the set of the leaf nodes in subtree  $V_v$  is denoted by  $U_v$ . All leaf nodes can be separated into two subsets, one is for information leaf nodes and the other is for frozen leaf nodes. As shown in Fig. 2(b), a node  $v$  in a tree is directly connected to a parent node  $p_v$ , a left child node  $v_l$  and a right child node  $v_r$ , respectively. The stage  $s$  of a node  $v$  is defined by the number of edges between node  $v$  and its nearest leaf node. All leaf nodes are at stage  $s = 0$ . The node  $v$ , which is not a leaf node, receives  $\alpha_v$  from its parent node, and generates  $\alpha_{v_l}$  according to (1) [7].



**Figure 2 : (a) Decoding architecture as a binary tree (b) Node v received/response information**

$$f_-: \alpha_{v_l}^i = \alpha_v^i \boxplus \alpha_v^{i+2^{s-1}}, i \in [0, 2^{s-1} - 1].$$

Node  $v$  sends the  $\alpha_{v_l}$  to its left child node and then waits for the feedback vector  $\beta_{v_l}$  to return. Subsequently, (2) [7] is used to calculate  $\alpha_{v_r}$  from  $\alpha_v$  and feedback vector  $\beta_{v_l}$ .

$$f_+: \alpha_{v_r}^i = (-1)^{\beta_{v_l}^i} \times \alpha_v^i + \alpha_v^{i+2^{s-1}}, i \in [0, 2^{s-1} - 1]$$

After receiving the feedback vector  $\beta_{v_r}$  from the right child, node  $v$  uses (3) to recover the feedback vector  $\beta_v$ , which is sent to its parent node  $p_v$ .

$$\begin{cases} \beta_v^i = \beta_{v_l}^i \oplus \beta_{v_r}^i \\ \beta_v^{i+2^{s-1}} = \beta_{v_r}^i \end{cases}, i \in [0, 2^{s-1} - 1]$$

The node  $v$ , which is a leaf node, receives  $\alpha_v$  from its parent node, and makes hard bit decision to get the feedback  $\beta_v$  directly. Thus, a leaf node is a bit-decision node. Both SC and SCL decoders can benefit from a series of multi-bit decision techniques to prune certain decoding tree branches.

### 3.3 SSC decoding

The simplification of the decoding process is an issue that should be prioritized from a practical standpoint. It was suggested that a simplified SSC (Simplified successive cancellation) decoder [31] could reduce the number of redundant calculations performed during SC decoding without having an impact on the performance of the error rate. As a result of the property of progressive polarization, how a compact-level code tree can be used to display polar codes just two levels are maintained because at this point all the channels have been polarized into their respective types at the third stage. The next two stages, with a combined total of four and eight levels each, are structured similarly. The code tree's nodes can all be categorized as either rate-zero, one or R nodes. It is possible that the progeny of these nodes is made up of a mixture of both frozen and informational bits. Encoder decoding activities are summarized here. Constructed codes with one or zero rates transmit hard messages directly, respectively; on the other hand, for rate-R nodes directly the hard messages are passed. The processing consists of conventional SC decoding techniques. The complexity has been decreased approximately 220 times as a result of the simplification in rate-one nodes and rate-zero nodes. This is in comparison to the original SC decoding.

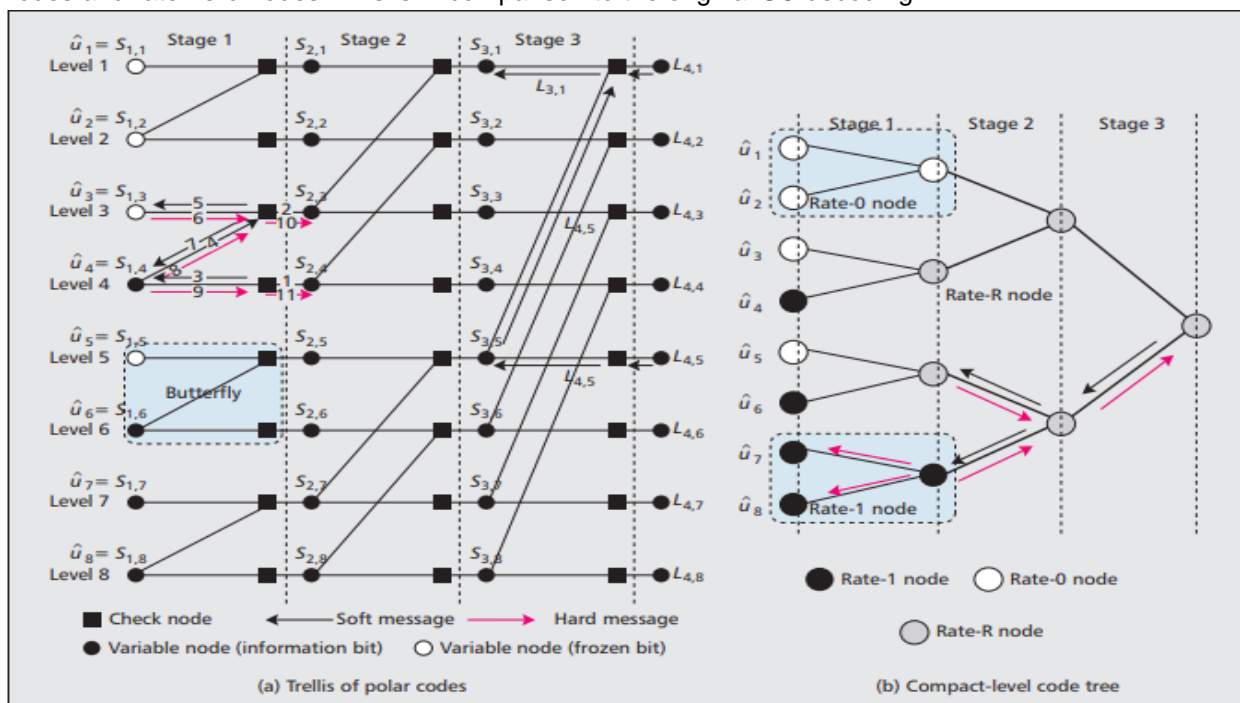


Figure 3 : Representation of SSC decoder

### 3.4 CA-SCL decoding

Techniques for SCL decoding that are enabled by CRC (cyclic redundancy checks), such as CA-SCL recently made public in [32] as a strategy for further improve the productivity of polar codes in various applications. In certain situations, the SCS/SCL decoder will run the alternative paths through a CRC detector, and the verification results will be utilized to determine which codeword is accurate. The paper proposes an adaptive CRC-aided SCL decoder, which is also known as a CA-SCL [33] as a method for lowering the SCL decoding computational complexity induced by a big list size by gradually increasing the list size. The performance of the polar codes has been improved greatly when using these CRC-aided decoding approaches, and it outperforms its performance when using ML decoding.

### 3.5 BP decoding

Arikan initially developed the concept of the Belief propagation decoder for polar codes in [34]. It uses in a manner similar to the lattice of polar code shown in Figure 1, the factor graph model is used to decode the message. Throughout the decoding process, the check node and the variable nodes will communicate with one another via soft messages. Hard messages are not exchanged at this point.

Therefore, the BP decoder can have significantly better performance than the Successive Cancellation decoder. On the other hand, Because of the cycles that are repeated on the factor graph, the message-passing schedule in BP is critical for channels other than BECs, and it is difficult to establish which schedule is the ideal one to use. In addition, the parallelism of the BP decoder is difficult to improve. Due of the higher implementation complexity and lower throughput of a BP decoder, it is less practical than a SC decoder.

The BP decoding of polar codes is a message passing algorithm, where information bits are retrieved by iterating over a factor graph corresponding to the generator matrix  $\mathbf{GN}$ . The calculate unit is referred to as processing elements (PE) [5], and it is shown below.  $L$  is the right-to-left message, and  $R$  is left-to-right message. The message propagation rules are as follows:

$$\begin{cases} L_{i,j-1} = g(L_{i+1,j}, L_{i+1,j+2^{n-i}} + R_{i,j+2^{n-i}}) \\ L_{i,j+2^{n-i}} = g(R_{i,j}, L_{i+1,j+2^{n-i}}) + L_{i+1,j+2^{n-i}} \\ R_{i+1,j} = g(R_{i,j+2^{n-i}} + L_{i+1,j+2^{n-i}}, R_{i,j}) \\ R_{i+1,j+2^{n-i}} = g(R_{i,j}, L_{i+1,j}) + R_{i,j+2^{n-i}} \end{cases}$$

Where  $g(x, y) = \text{sign}(x)\text{sign}(y) \min(|x|, |y|)$ ,  $\text{sign}(x)$  represents the sign of  $x$ , and  $i$  is denoted the stage number and  $j$  denotes the number of PE in a stage counting from top to bottom. Because the Tanner graph is based on no bit-reversal permutation matrix, so the message propagation rules is more concise.  $L, R$  messages are changed once a iteration, and they are initialized as follows:

$$L_{i,j} = \begin{cases} 0, & j \neq n+1 \\ LLR_i & j = n+1 \end{cases}$$

$$R_{i,j} = \begin{cases} 0, & \forall j, i \in \mathcal{A} \\ +\infty, & j = 1, i \in \mathcal{A}^c \end{cases}$$

We get  $LLR_i$  from the  $i$ -th received bit, and Channel condition. Every iteration the hard information is decided as follows:

$$\hat{u}_i = \begin{cases} 0, & L_{1,i} + R_{1,i} \geq 0 \\ 1, & L_{1,i} + R_{1,i} < 0 \end{cases}$$

$$\hat{x}_i = \begin{cases} 0, & L_{n+1,i} + R_{n+1,i} \geq 0 \\ 1, & L_{n+1,i} + R_{n+1,i} < 0 \end{cases}$$

where,  $\hat{x}^i$  is the estimation of  $x^i$ ,  $\hat{u}^i$  is the estimation of  $u^i$ .

### 3.6 SCAN method

However, even though the BP decoder has better performance than the Successive Cancellation decoder and produces softer outputs, the processing complexity and space requirements are extremely high. In contrast, the SC decoder lacks these advantages. Decoding polar codes without soft outputs, such as those used in turbo-based receivers, is difficult with polar codes, hence the Successive Cancellation decoder remains an alternative for low-cost decoding [35]. We want to make it possible to use polar codes in situations where soft-output decoders are required. The soft cancellation (SCAN) decoder, a different version of the Successive Cancellation decoder that has limited capability and soft output, is one such example. This decoder provides information on the consistency of encoded bits as well as bits that have not been encrypted, thus it can be used in a variety of situations. Much of the complexity can be reduced by using SCAN. For instance, in order to obtain the same FER effectiveness across a diode channel, the SCAN decoder just requires two iterations, in contrast to the BP decoder, which requires sixty iterations. Additionally, the Soft cancellation decoder outperforms the BP decoder as the number of iterations is increased even further. In addition, the Soft cancellation decoder only needs  $5N^2 + N \log_2 N^2$  memory elements, which is a considerable reduction over the memory requirements of the BP decoder, which are  $2N(\log_2 N + 1)$  memory elements.

## 4. Results and discussion

Polar codes have garnered a great deal of attention due to their capability of reaching Shannon's capacity with an effective successive cancellation (SC) decoder [27]. Over an AWGN channel, we will use Binary Phase Shift Keying, abbreviated BPSK. The BER and FER of SC decoder with  $N = 256$  and  $L = 128$  is given in Figure 4.



The number of errors that occur in a given amount of time is referred to as the bit error rate (BER). The bit error ratio, often known as BER, is calculated by dividing the number of bit errors that occurred over a period of time under study by the total amount of bits that were conveyed.

The ratio of data that was received with errors to the total data that was received is known as the Frame Error Rate (FER). Utilized for the purpose of evaluating the quality of a signal link. It is possible that the connection will be severed if the FER is too high (if there are too many errors).

In figure 4, the value of N is 256 and the value of L is 128. Both BER and FER is considered for the polar code, SC. It can be seen that the SC attains a BER of 0.2 at an SNR of 1 dB and a FER of 0.5 at an SNR of 1 dB. As the SNR increases, the BER and FER reduces. The BER and FER of BPSK is also given. To modulate the bits, a Binary Phase Shift Keying (BPSK) modulation is used.

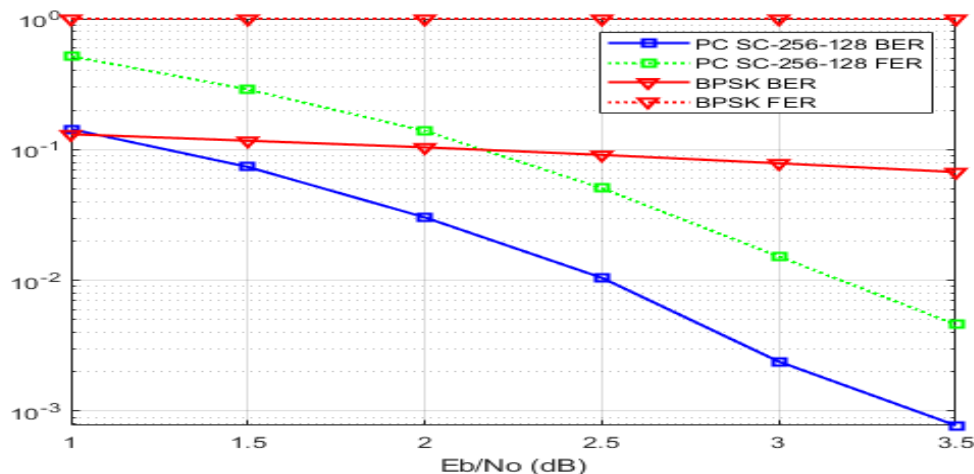


Figure 4 : BER and FER for Successive Cancellation technique

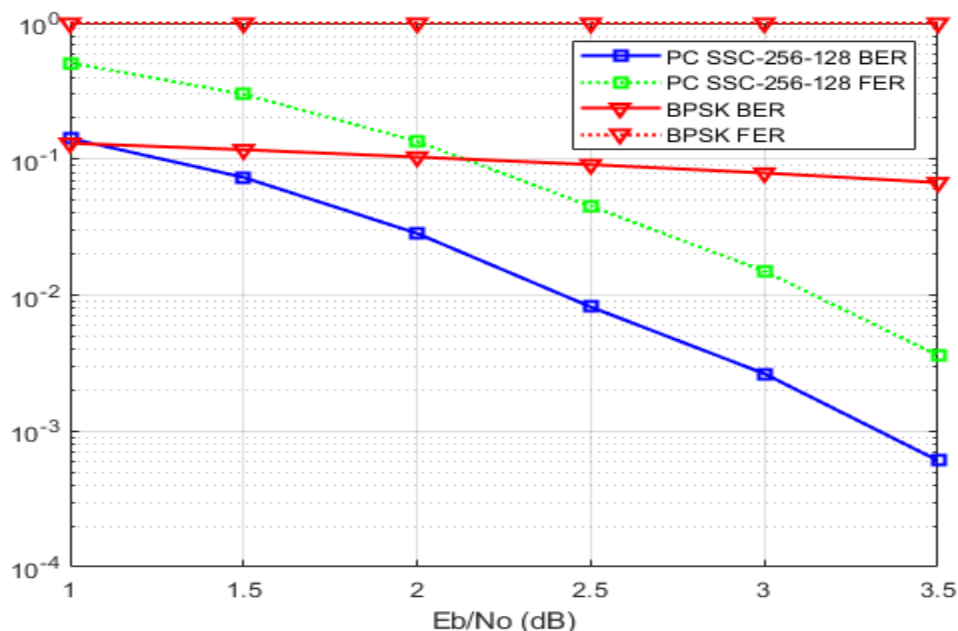


Figure 5 : BER and FER for SSC

Figure 5 gives the BER and FER of SSC decoder. For SSC decoder, the value of N is 256 and the value of L is 128. Both BER and FER is considered for the polar code, SSC. It can be seen that the SC attains a BER of 0.2 at an SNR of 1 dB and a FER of 0.4 at an SNR of 1 dB. As the SNR increases, the BER and FER reduces. The BER reaches a value of 0.006 at an SNR of 3.5 dB. The



BER and FER of BPSK is also given. To modulate the bits, a Binary Phase Shift Keying (BPSK) modulation is used.

Figure 6 gives the BER and FER of BP decoder. The BP decoder is utilizing a 40 bit CRC. The BER and FER is considered for various values of N and L. With N = 256 and L = 128, The BER is lower at a SNR of 2 dB when compared to other combinations of N and L while it is higher with a value of 0.02 at a SNR of 33 dB when compared to other combinations of N and L. A BP decoder with N = 1024 and L = 512 is having the lowest BER of 0.004 at SNR of 3.5 dB.

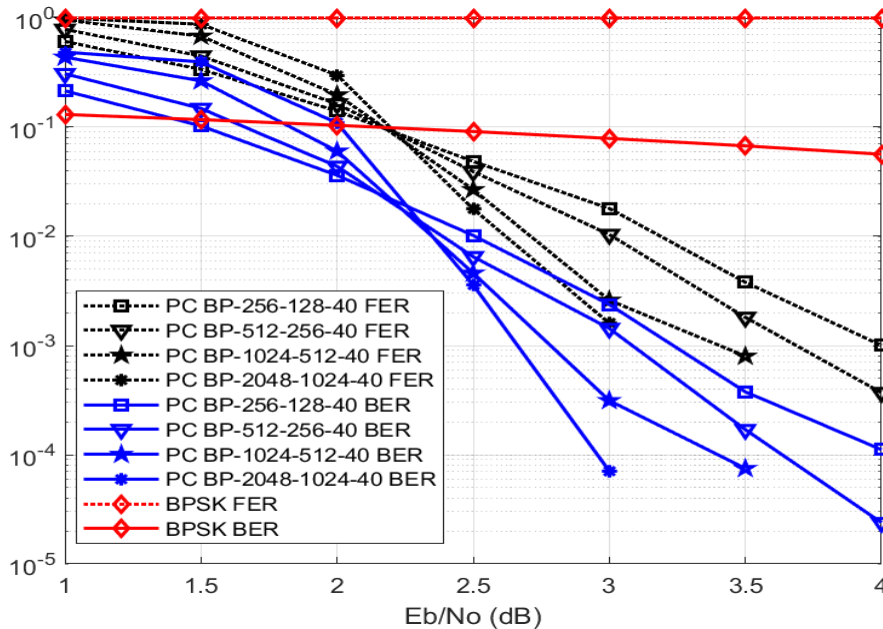


Figure 6 : BER and FER for BP decoder

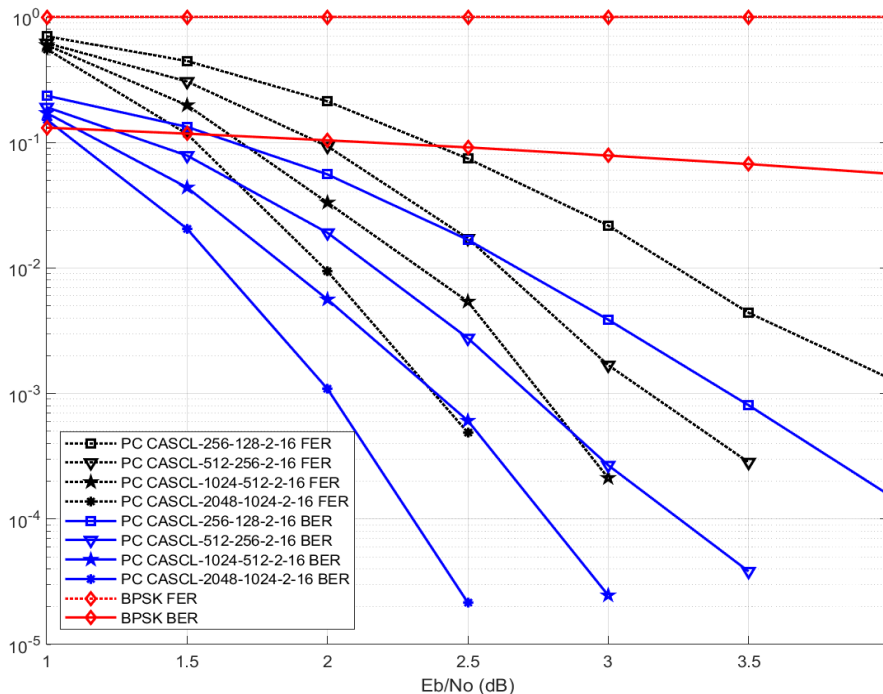


Figure 7 : BER and FER for CA-SCL

Figure 7 gives the BER and FER of CA-SCL decoder and figure 8 gives the BER and FER of SCAN decoder.

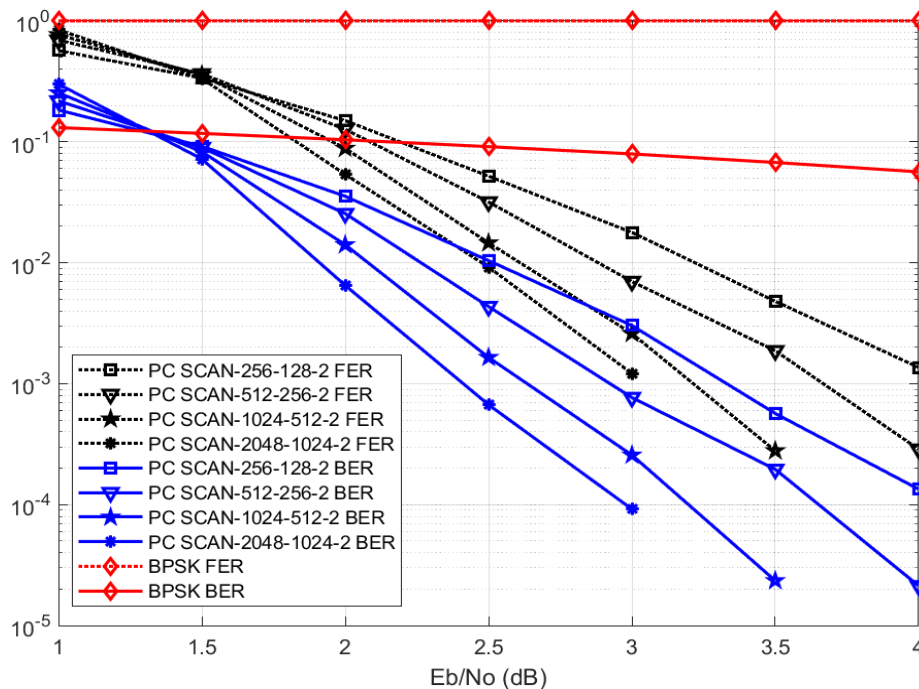


Figure 8 : BER and FER for SCAN

## 5. Conclusion

This research assessed the performance of 5G channel coding methods for short and moderate information lengths and described the polar code encoding and decoding processes used in wireless system standards. In the proposed research, an extensive work on the existing technologies to show that coding technique is capacity-achieving using different channel models. The decoders, Successive Cancellation (SC), Simplified Successive Cancellation (SSC), Cyclic Redundancy Check-Aided SCL (CA SCL), Belief Propagation (BP) decoder, and Soft Cancellation (SCAN) are considered. The Bit Error Rate (BER) and Frame Error Rate (FER) of these decoders for various bits are considered.

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