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# Optimal Solution of Intuitionistic Fuzzy Transportation Problem Using Intuitionistic Triangular Fuzzy Numbers

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**Abstract**: The fuzzy transportation problem is a valuable tool for decision-makers facing real-world transportation scenarios with uncertain data, helping them make more robust and realistic decisions. To deal with situations where both degrees of membership and non-membership are to be considered, intuitionistic fuzzy is taken into account. This paper deals with an intuitionistic fuzzy transportation problem whose boundaries are intuitionistic three-sided fuzzy numbers. In the current work, a proficient strategy is proposed to tackle the intuitionistic fuzzy transportation problem in which all the boundaries are taken as three-sided intuitionistic fuzzy numbers. A mathematical model is given to describe the strategy. The validity of the proposed algorithm is demonstrated by a numerical example. The proposed algorithm is easy to implement and provides an optimal solution to the intuitionistic transportation problem.

**Keywords**: Intuitionistic fuzzy transportation problem, intuitionistic triangular fuzzy number.

## 1. Introduction

Transportation problem is a specific class of linear programming problem which is co-related with day-to-day activities in our whole life. In this transportation problem, the commodities are transported from a numerous set of sources to a numerous set of destination to meet the particular requirements. The major target of transportation problem is to meet the demand at destinations from the supply sources at the least transportation cost.

Zadeh [10] suggested the concept of decision making in fuzzy environment. A fuzzy transportation problem consists of cost, supply and demand which are represented by fuzzy numbers. The main purpose of fuzzy transportation problem is to minimize the total fuzzy transportation cost which satisfies fuzzy supply and fuzzy demand.

Atanassov [1] proposed the concept of intuitionistic fuzzy set in 1986 which is mostly require dealing with uncertainty as an extension of fuzzy set theory. Intuitionistic fuzzy transportation problem is a problem in which the supply and demand are in intuitionistic fuzzy quantities. In intuitionistic fuzzy set, the main objective of intuitionistic fuzzy set is that it isolates the both degree of membership and degree of non-membership of an element in the set. The sum of membership and non-membership always lie between [0,1].

Sudhakar et al. [8] studied an approach for finding an optimal solution for transportation problem using zero suffix method. Antony et al. [2] solved the fuzzy transportation problem using Vogel's approximation method. Thamaraiselvi and Santhi [9] described the intuitionistic fuzzy transportation problem using hexagonal intuitionistic fuzzy numbers for finding the both feasible and optimal solution. NagoorGani and Abbas [7] proposed a Monalisha's approximation method (MAM) for solving the intuitionistic fuzzy transportation problem.

Edward et al. [4] developed a new technique for solving unbalanced intuitionistic fuzzy transportation problem to obtain the optimal solution whose parameters are represented by intuitionistic triangular fuzzy number. Josephine et al. [6] proposed a dynamic method for solving intuitionistic fuzzy transportation problem in which all the parameters are trapezoidal fuzzy number.

Fathy [5] introduced a method to solve an interval-valued intuitionistic fuzzy linear programming by splitting it into nine crisp linear programs and then used additional bounded constraint for solving crisp linear programs. Dhanasekar et al. [3] solved the interval-valued trapezoidal intuitionistic fuzzy transportation problem using score expected function.

The motivation towards the study is to develop an algorithm to deal with such transportation problems that uses the intuitionistic three sided fuzzy numbers. The algorithm is easy to handle, uses less number of iterations and gives optimal solution of the problem.

The paper is organized as follows: Section 2 contains basic definitions used in the manuscript. Arithmetic operations and ranking of triangular intuitionistic fuzzy number is given in Section 3. Mathematical formulation of intuitionistic fuzzy transportation problem is presented in Section 4. The proposed algorithm to solve intuitionistic fuzzy transportation problem is presented in Section 5. A numerical example depicting the validation of proposed algorithm is given in Section 6. Concluding remarks are given in Section 7.

## 2. Preliminaries

Some basic definitions are presented in this section.

**Definition 2.1 [7]:** An Intuitionistic fuzzy set $\tilde{A}^I$ in X is a collection of ordered pairs$\tilde{A}^I$ = $\{< z, \mu_{\tilde{A}^I}(z)$, $\nu_{\tilde{A}^I}(z) >$: z∈ $X\}$, where$\mu_{\tilde{A}^I}(z)$,$\nu_{\tilde{A}^I}(z)$: X →[0,1] are the functions such that $0 \leq \mu_{\tilde{A}^I}(z), +\nu_{\tilde{A}^I}(z) \leq 1 \forall$ z∈ $X$. For each z of the numbers $\mu_{\tilde{A}^I}(z)$represents the membership function and $\nu_{\tilde{A}^I}(z)$represents the non-membership function.

Intuitionistic fuzzy set is indicated by $\tilde{A}^I$.

**Definition 2.2 [7]:** An intuitionistic fuzzy subset $\tilde{A}^I$ = $\{< z, \mu_{\tilde{A}^I}(z)$,$\nu_{\tilde{A}^I}(z)$: z ∈ $X\}$of the real line R is called an intuitionistic fuzzy number if the following conditions holds:

1. There exists an element r ∈ R, $\mu_{\tilde{A}^I}(r)$= 1 and $\nu_{\tilde{A}^I}(r)$= 0.
2. $\mu_{\tilde{A}^I}(z)$is a continuous mapping from R to the closed interval [0,1] and ∀z ∈ R, the relation $0 \leq \mu_{\tilde{A}^I}(z) + \nu_{\tilde{A}^I}(z) \leq 1$ holds.

The membership and non-membership function of $\tilde{A}^I$ is of following form:

$$\mu_{\tilde{A}^I}(z)=\begin{cases} 0 & for -\infty < z \leq r-\alpha_1 \\ p_1(z) & for\ z\,\epsilon\,[r-\alpha_1, r] \\ 1 & for\ z = r \\ q_1(z) & for\ z\,\varepsilon\,[r, r+\beta_1] \\ 0 & for\ r+\beta_1 \leq z < \infty \end{cases}$$

Where $p_1(z)$and $q_1(z)$ are strictly increasing and decreasing function in [r-$\alpha_1$, r] and [r, r+$\beta_1$] respectively.

$$\nu_{\tilde{A}^I}(z)=\begin{cases} 0 & for -\infty < z \leq r-\alpha_2 \\ p_2(z) & for\ z\,\epsilon\,[r-\alpha_2, r] \\ 1 & for\ z = r \\ q_2(z) & for\ z\,\varepsilon\,[r, r+\beta_2] \\ 0 & for\ r+\beta_2 \leq z < \infty \end{cases}$$

Where m is the mean value of $\tilde{A}^I$, $\alpha_1$ is left spread and $\beta_1$is right spread of membership function$\mu_{\tilde{A}^I}(z)$. $\alpha_2$ is left spread and $\beta_2$is right spread of non-membership function $\nu_{\tilde{A}^I}(z)$.

**Definition 2.3 [7]:** A triangular intuitionistic fuzzy number $\tilde{A}^I$ is an intuitionistic fuzzy set in R with the membership function $\mu_{\tilde{A}^I}(z)$and non – membership function $\nu_{\tilde{A}^I}(z)$defined by

$$\mu_{\tilde{A}^I}(z)=\begin{cases} \frac{z-z_1}{z_2-z_1} & for\ z_1 \leq z \leq z_2 \\ \frac{z_3-z}{z_3-z_2} & for\ z_2 \leq z \leq z_3 \\ 0 & otherwise \end{cases} \quad \nu_{\tilde{A}^I}(z)=\begin{cases} \frac{z_2-z}{z_2-z_1'} & for\ z_1' \leq z \leq z_2 \\ \frac{z-z_1}{z_3'-z_2} & for\ z_2 \leq z \leq z_3' \\ 1 & otherwise \end{cases}$$

where $z_1' \leq z_1 \leq z_2 \leq z_3 \leq z_3' \ \forall$ z ∈ X.

## 3. Arithmetic operations and ranking of triangular intuitionistic fuzzy number [7]

Let $\tilde{X}^I$ = $(x_1, x_2, x_3; x_1', x_2, x_3')$ and $\tilde{Y}^I$ = $(y_1, y_2, y_3; y_1', y_2, y_3')$ be two triangular fuzzy numbers. Then

- **Addition**: $(x_1, x_2, x_3; x_1', x_2, x_3') + (y_1, y_2, y_3; y_1', y_2, y_3') = (x_1+y_1,\ x_2+y_2,\ x_3+y_3;\ x_1'+y_1',\ x_2+y_2,\ x_3'+y_3')$

- **Subtraction**: $(x_1, x_2, x_3; x_1', x_2, x_3') - (y_1, y_2, y_3; y_1', y_2, y_3')$ = $(x_1 - y_3,\ x_2 - y_2,\ x_3 - y_1;\ x_1' - y_3',\ x_2 - y_2,\ x_3' - y_1')$

- **Multiplication**: $(x_1, x_2, x_3; x_1', x_2, x_3') \otimes (y_1, y_2, y_3; y_1', y_2, y_3')$ = $(x_1y_1,\ x_2y_2,\ x_3y_3;\ x_1'y_1',\ x_2y_2,\ x_3'y_3')$

- **Scalar Multiplication**: k$(x_1, x_2, x_3; x_1', x_2, x_3')$ = (k$x_1$, k$x_2$, k$x_3$; k$x_1'$,k$x_2$, k$x_3'$)

**Ranking of triangular intuitionistic fuzzy number**

If $\tilde{A}^I$ = $(x_1, x_2, x_3; x_1', x_2, x_3')$ is a triangular fuzzy number then it is defined as,

$\tilde{A}^I = \frac{x_1+3x_2+x_3+x_1'+3x_2+x_3'}{6}$.

**4. Mathematical formulation of intuitionistic fuzzy transportation problem**

Assume a transportation problem with m (say) intuitionistic fuzzy sources and n (say) intuitionistic fuzzy destinations. Allow $C_{ij}$ to be the expense of moving the item structure sources to destinations. Let $x_i$ and $y_j$ be the amount of items accessible at intuitionistic fuzzy beginning and intuitionistic fuzzy destination individually. Allow $z_{ij}$to be the amount transport from intuitionistic fuzzy sources to intuitionistic fuzzy destinations.

The statistical model of intuitionistic fuzzy transportation problem is as per the following:

Minimize $\tilde{Z}^I \approx \sum_{i=1}^{m}\sum_{j=1}^{n} \tilde{c}^I{}_{ij}\, \tilde{z}^I{}_{ij}$

Subject to $\sum_{j=1}^{m} \tilde{z}^I{}_{ij} \approx \tilde{x}^I{}_i$; i = 1,2,…,m

$\sum_{i=1}^{n} \tilde{z}^I{}_{ij} \approx \tilde{y}^I{}_j$; j= 1,2,…,n (1)

$\tilde{z}^I{}_{ij} \geq 0$ for i = 1,2,…,m, j = 1,2,…,n.

**5. Algorithm for solving intuitionistic fuzzy transportation problem**

Step 1: Defuzzify the boundaries for the specific problem. It is mostly concentrated on the integers and also occurrence of no integer rounding off the integers is substantial.

Step 2: Choose the smallest odd cost from the table and subtract that odd cost throughout the odd cost from the matrix. Thus, if atleast one zero holds and persist all cost become even.

Step 3: If there exist any zero, assign the minimum supply or demand.

Step 4: After that, multiply ½ by each column.

Step 5: Now, after observing the minimum odd cost from row or column exempting zeros in column.

Step 6: By adding up the product of cost and value placed in demand or supply then total minimum cost of intuitionistic fuzzy transportation problem will be calculated.

Step 7: To find an optimal solution, proceeding from step 2, step 3 and step 4 till an optimal solution is reached.

**6. Numerical example to solve the intuitionistic fuzzy transportation problem**

Consider an intuitionistic fuzzy transportation problem whose quantities are triangular intuitionistic fuzzy numbers. The transportation cost per unit when an item is transported, is given in the following tabular form.

| | $D_1$ | $D_2$ | $D_3$ | Supply |
|---|---|---|---|---|
| $O_1$ | (1,2,3;-8,2,10) | (2,6,8;5,6,9) | (0,1,2;-2.1,6) | (3,4,9;2,4,10) |
| $O_2$ | (6,7,9;4,7,11) | (11,13,15;10,13,18) | (6,8,12;5,8,13) | (7,9,10;8,9,11) |
| $O_3$ | (3,5,6;2,5,7) | (3,6,11;4,6,12) | (8,9,10;7,9,11) | (6,10,14;8,10,14) |

| | | | | |
|---|---|---|---|---|
| Demand | (10,11,14;8,11,16) | (9,11,14;10,11,15) | (4,5,7;3,5,8) | |

Solution: To find the solution of above transportation problem, the steps of proposed algorithm are followed.
Step 1: Applying the ranking function and defuzzification, the following integers are obtained.

$x_{11}$= 3; $x_{12}$ = 10; $x_{13}$ = 2
$x_{21}$ = 12; $x_{22}$= 22; $x_{23}$ = 14 (2)
$x_{31}$ = 8; $x_{32}$ = 11; $x_{33}$ = 15

**Table 1: On defuzzification, the cost from the given table using ranking function:**

| | $D_1$ | $D_2$ | $D_3$ | Supply |
|---|---|---|---|---|
| $O_1$ | 3 | 10 | 2 | (3,4,9;2,4,10) |
| $O_2$ | 12 | 22 | 14 | (7,9,10;8,9,11) |
| $O_3$ | 8 | 11 | 15 | (6,10,14;8,10,14) |
| Demand | (10,11,14;8,11,16) | (9,11,14;10,11,15) | (4,5,7;3,5,8) | |

Step 2: Select the smallest odd cost from the above table and subtract that odd cost throughout the odd cost from the matrix. Thus, if at least one zero holds and persist all cost become even. So, from the above table, minimal odd cost is 3 and now subtracts 3 from the entire odd cost matrix.

**Table 2: Updated table using step 2:**

| | $D_1$ | $D_2$ | $D_3$ | Supply |
|---|---|---|---|---|
| $O_1$ | 0 | 10 | 2 | (3,4,9;2,4,10) |
| $O_2$ | 12 | 22 | 14 | (7,9,10;8,9,11) |
| $O_3$ | 8 | 8 | 12 | (6,10,14;8,10,14) |
| Demand | (10,11,14;8,11,16) | (9,11,14;10,11,15) | (4,5,7;3,5,8) | |

Step 3: If there exist any zero, assign the minimum supply or demand. In the first row, one cost is zero then assign the supply quantity and then delete the first row.

**Table 3: Updated table using step 3:**

| | $D_1$ | $D_2$ | $D_3$ | Supply |
|---|---|---|---|---|
| $O_1$ | 0(3,4,9;2,4,10) | 10 | 2 | (3,4,9;2,4,10) |
| $O_2$ | 12 | 22 | 14 | (7,9,10;8,9,11) |
| $O_3$ | 8 | 8 | 12 | (6,10,14;8,10,14) |
| Demand | (10,11,14;8,11,16) | (9,11,14;10,11,15) | (4,5,7;3,5,8) | |

Step 4: All costs are even, so multiply by ½ with all costs. In the above table, even costs are left after deleting the first row then multiply by ½ with all costs.

**Table 4: Updated table using step 4:**

| | $D_1$ | $D_2$ | $D_3$ | Supply |
|---|---|---|---|---|
| $O_2$ | 6 | 11 | 7 | (7,9,10;8,9,11) |
| $O_3$ | 4 | 4 | 6 | (6,10,14;8,10,14) |
| Demand | (1,8,11;2,8,14) | (9,11,14;10,11,15) | (4,5,7;3,5,8) | |

Repeat the same process until an optimal solution is obtained.
Step 5: Using step 2, Select the smallest odd cost from the above table and subtract that odd cost throughout the odd cost from the matrix. Thus, if at least one zero holds and persist all cost become even. In the above table, minimal odd cost is 7 and then subtracts 7 from the all odd costs matrix.

**Table 5: Updated table using step 5:**

| | $D_1$ | $D_2$ | $D_3$ | Supply |
|---|---|---|---|---|
| $O_2$ | 6 | 4 | 0 | (7,9,10;8,9,11) |
| $O_3$ | 4 | 4 | 6 | (6,10,14;8,10,14) |
| Demand | (1,8,11;2,8,14) | (9,11,14;10,11,15) | (4,5,7;3,5,8) | |

Step 6: Using step 3, If there exist any zero, assign the minimum supply or demand. In the third column, one cost is zero then assign the demand quantity and then delete the third column.

**Table 6: Updated table using step 6:**

| | $D_1$ | $D_2$ | $D_3$ | Supply |
|---|---|---|---|---|
| $O_2$ | 6 | 4 | 0(4,5,7;3,5,8) | (7,9,10;8,9,11) |
| $O_3$ | 4 | 4 | 6 | (6,10,14;8,10,14) |
| Demand | (1,8,11;2,8,14) | (9,11,14;10,11,15) | (4,5,7;3,5,8) | |

Step 7: Using step 4, all costs are even, multiply ½ by each column. In the above table, even costs are left after deleting the third column then multiply by ½ with all costs.

**Table 7: Updated table using step 7:**

| | $D_1$ | $D_2$ | Supply |
|---|---|---|---|
| $O_2$ | 3 | 2 | (0,4,6;0,4,8) |
| $O_3$ | 2 | 2 | (6,10,14;8,10,14) |
| Demand | (1,8,11;2,8,14) | (9,11,14;10,11,15) | |

Step 8: Again by step 2, select the smallest odd cost from the above table and subtract that odd cost throughout the odd cost from the matrix. Thus, if at least one zero holds and persist all cost become even. In the above table, minimal odd cost is 3 and then subtracts 3 from the all odd cost matrix.

**Table 8: Updated table using step 8:**

| | $D_1$ | $D_2$ | Supply |
|---|---|---|---|
| $O_2$ | 0 | 2 | (0,4,6;0,4,8) |

| | | | |
|---|---|---|---|
| $O_3$ | 2 | 2 | (6,10,14;8,10,14) |
| Demand | (1,8,11;2,8,14) | (9,11,14;10,11,15) | |

Step 9: Again by step 3, If there exist any zero, assign the minimum supply or demand. In the second row, one cost is zero then assign the supply quantity and then delete the second row.

**Table 9: Updated table using step 9:**

| | $D_1$ | $D_2$ | Supply |
|---|---|---|---|
| $O_2$ | 0(0,4,6;0,4,8) | 2 | (0,4,6;0,4,8) |
| $O_3$ | 2 | 2 | (6,10,14;8,10,14) |
| Demand | (-5,4,11;-6,4,14) | (9,11,14;10,11,15) | |

Step 10: Again by step 4, all costs are even hence multiply by ½ by each column. In above table, only third row is left in which all costs are even then multiply by ½ by each column.

**Table 10: Updated table using step 10:**

| | $D_1$ | $D_2$ | Supply |
|---|---|---|---|
| $O_3$ | 1 | 1 | (6,10,14;8,10,14) |
| Demand | (-5,4,11;-6,4,14) | (9,11,14;10,11,15) | |

Step 11: Again Using step 2, Select the smallest odd cost from the above table and subtract that odd cost throughout the odd cost from the matrix. Thus, if at least one zero holds and persist all cost become even. In above table, all costs are odd and both cost value is 1 then subtract 1 itself from both costs.

**Table 11: Updated table using step 11:**

| | $D_1$ | $D_2$ | Supply |
|---|---|---|---|

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| | | | |
|---|---|---|---|
| $O_3$ | 0 | 0 | (6,10,14;8,10,14) |
| Demand | (-5,4,11; -6,4,14) | (9,11,14;10,11,15) | |

Step 12: By step 5, observing the minimum odd cost from row or column exempting zeros in the column. Assign the demand quantity in first column and then delete that column.

**Table 12: Updated table using step 12:**

| | $D_1$ | $D_2$ | Supply |
|---|---|---|---|
| $O_3$ | 0(-5,4,11;-6,4,14) | 0 | (6,10,14;8,10,14) |
| Demand | (-5,4,11;-6,4,14) | (9,11,14;10,11,15) | |

Step 13: Give the original cost to that cost which has zero cost.

**Table 13: Updated table using step 13:**

| | $D_1$ | $D_2$ | $D_3$ | Supply |
|---|---|---|---|---|
| $O_1$ | 3(3,4,9;2,4,10) | 10 | 2 | (3,4,9;2,4,10) |
| $O_2$ | 12(0,4,6;0,4,8) | 22 | 14(4,5,7;3,5,8) | (7,9,10;8,9,11) |
| $O_3$ | 8(-5,4,11;-6,4,14) | 11 | 15 | (6,10,14;8,10,14) |
| Demand | (10,11,14;8,11,16) | (9,11,14;10,11,15) | (4,5,7;3,5,8) | |

Step 14: Using step 6, Specified all the parameters from given table then adding up the product of cost and value placed in demand or supply total minimum cost of intuitionistic fuzzy transportation problem.

**Table 14: Updated table using step 14:**

| | $D_1$ | $D_2$ | $D_3$ | Supply |
|---|---|---|---|---|
| $O_1$ | 3(3,4,9;2,4,10) | (2,6,8;5,6,9) | (0,1,2;-2,1,6) | (3,4,9;2,4,10) |
| $O_2$ | 12(0,4,6;0,4,8) | (11,13,15;10,13,18) | 14(4,5,7;3,5,8) | (7,9,10;8,9,11) |

| | | | | |
|---|---|---|---|---|
| $O_3$ | 8(-5,4,11;-6,4,14) | (3,6,11;4,6,12) | (8,9,10;7,9,11) | (6,10,14;8,10,14) |
| Demand | (10,11,14;8,11,16) | (9,11,14;10,11,15) | (4,5,7;3,5,8) | |

Using step 7, the optimal solution is
Minimize Z = 3(3,4,9;2,4,10) + 12(0,4,6;0,4,8) + 8(-5,4,11;-6,4,14) + 14(4,5,7;3,5,8)
= (9,12,27;6,12,30) + (0,48,72;0,48,96) + (-40,32,88;-48,32,112)
= (9,60,99;6,60,126) + (16,102,186;-6,102,224)
= (25, 162,285; 0,162,350)

## 7. Conclusion

An algorithm is proposed to obtain the best (optimal) solution of fuzzy transportation problem in which all the supply and demand quantities has been taken as intuitionistic three sided fuzzy numbers. A numerical example is provided for the execution of the algorithm. The method is easier to implement and it is considered to require the minimum number of iterations to acquire the optimal solution.

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